Scientific
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Novel Spectrometer –
The Linear Dispersion Compound Prism Spectrometer

ABSTRACT

Prism based spectrometers are utilizing the wavelength dispersive characteristics of the prism. Their main advantage over grating based spectrometers is the lack of high-order phenomena which lead to ghosts and cross-talk. But the main limit of prisms was always the non-linear behavior of angle vs. wavelength. This complicates the use prisms with CCD or CMOS detectors while retaining constant wavelength resolution or SNR, and thus limits the existence of prism based spectrometers and spectral imagers.

The present research describes a method and optical design for a linear spectral dispersion, by using a compound prism.

Keywords: Concept, Optical design, Linear Spectral Dispersion, Compound Prism

1. Introduction

Research essentials are presented on the following pages.

Introduction, conclusions, and references - will be completed soon.

Dispersion in a prism

The angular deviation of a thin prism may be approximated as

(3.1) $$D = (n-1)\cdot\theta$$

where n is the refractive index and θ is the apex angle of the prism. The derivative is the angular dispersion:

(3.2) $$\mathrm{dD} = \mathrm{D}\frac{\mathrm{dn}}{\mathrm{n}-1}$$

Fig. 1 shows the angular dispersion of a prism made of BK7 glass. If we define our spectral region of interest in terms of λ_{min}, λ_{mid} and λ_{max}, we mark the refractive index at λi as $n_{\lambda i}$ and we can define the dispersion as:

(3.3) $$dn = n_{\lambda\max} - n_{\lambda\min}$$

and the Abbe number as

(3.4) $$v = \frac{n_{\lambda mid} - 1}{n_{\lambda\max} - n_{\lambda\min}} = \frac{n_{\lambda mid} - 1}{dn}$$

and we can derive a simpler expression for the angular dispersion:

(3.5) $$\mathrm{dD} = \frac{\mathrm{D}}{\mathrm{v}}$$

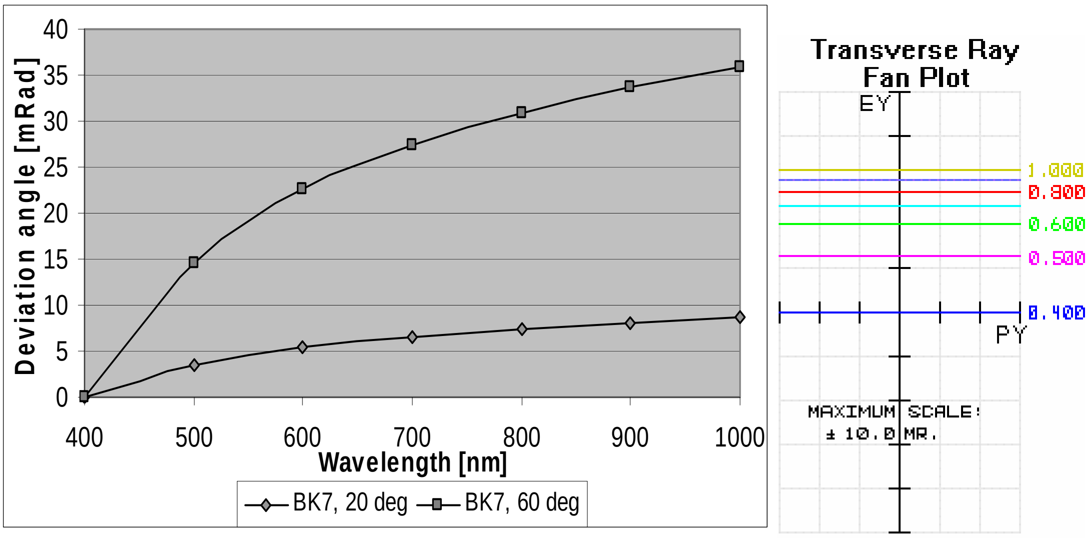

Fig 1: Angular dispersion in a prism. The change is not linear to the wavelength. Deviation angles are calculated relative to the deviation at 400nm.

Compound prism

When combining two prisms with opposite orientation (a compound prism), as in fig. 2, the total angular deviation may be approximated by:

(3.6) $$D = D_1 + D_2$$

And the total angular dispersion is:

(3.7) $$dD = dD_1 + dD_2$$

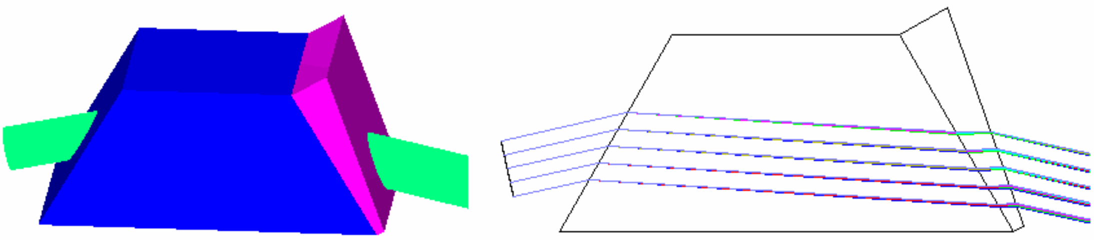

Fig 2: A compound prism, solid and ray trace view

In order to minimize the angular dispersion we want:

(3.8) $$dD = \frac{D_1}{v_1} + \frac{D_2}{v_2} = 0$$

Simple algebra would give:

(3.9) $$D_1 = \frac{Dv_1}{v_1 - v_2} \text{ and } D_2 = \frac{Dv_2}{v_2 - v_1}$$

And also:

(3.10) $$\theta_1 = \frac{Dv_1}{(n_{\lambda mid,1} - 1)(v_1 - v_2)} \text{ and } \theta_2 = \frac{Dv_2}{(n_{\lambda mid,2} - 1)(v_2 - v_1)}$$

Where θ_i are the apex angles of the prisms, as previously defined.

Secondary angular dispersion

The doublet configuration is limited and provides equal angular dispersion for only two wavelengths, namely λ_{min} and λ_{max} as can be seen in fig. 3.

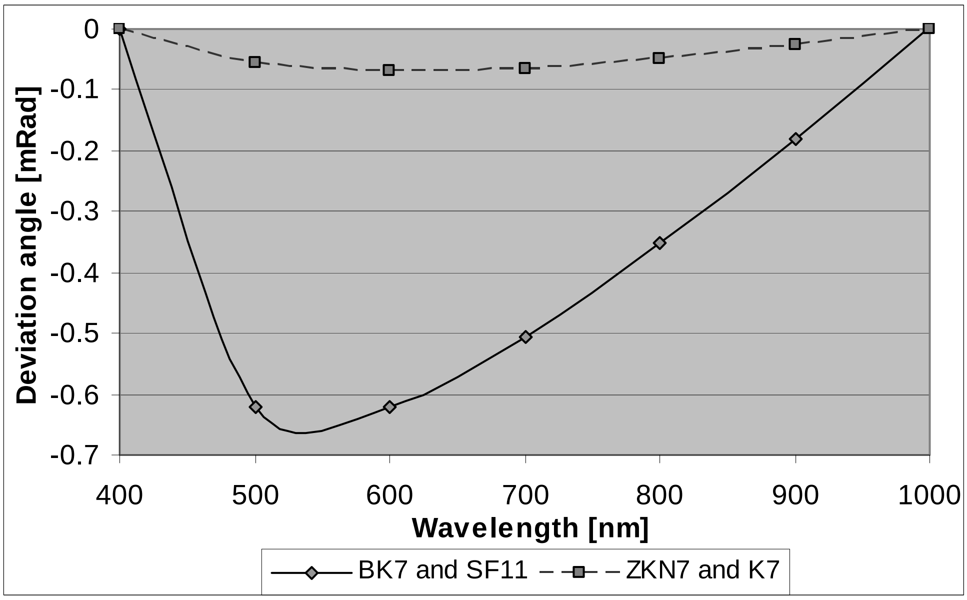

Fig 3: Different pairs of glass materials will have different secondary angular dispersion. Deviation angles are calculated relative to the deviation at 400nm.

In order to calculate the secondary angular dispersion at wavelength λmid we will use the partial dispersion coefficient:

(3.11) $$p = \frac{n_{\lambda\max} - n_{\lambda mid}}{n_{\lambda\max} - n_{\lambda\min}}$$

To calculate the secondary angular dispersion we substitute v with v/p since

(3.12) $$\frac{v}{p} = \frac{n_{\lambda mid} - 1}{n_{\lambda\max} - n_{\lambda\min}} \Big/ \frac{n_{\lambda\max} - n_{\lambda mid}}{n_{\lambda\max} - n_{\lambda\min}} = \frac{n_{\lambda mid} - 1}{n_{\lambda\max} - n_{\lambda mid}} = v_{partial}$$

and thus based on (3.5) and (3.7), and marking the secondary dispersion as dD3,

(3.13) $$dD3 = \frac{D_1 \cdot p_1}{v_1} + \frac{D_2 \cdot p_2}{v_2}$$

We now substitute D_1 and D_2 from (3.9)

(3.14) $$dD3 = \frac{Dv_1 p_1}{v_1(v_1 - v_2)} + \frac{Dv_2 p_2}{v_2(v_2 - v_1)} = -\frac{Dp_1}{(v_2 - v_1)} + \frac{Dp_2}{(v_2 - v_1)} = \frac{D(p_2 - p_1)}{(v_2 - v_1)}$$

and we have

(3.15) $$dD3 = \frac{D \cdot \Delta p}{\Delta v}$$

When designing an achromatic doublet we want dD3 to be minimal, so we try to find two glass materials such that Δp would be minimal and Δv would be maximal. We create a p-v plot for all the glass materials that may be used. Such a plot may be seen in fig. 4. It should be noted that the plot depends on the wavelengths we choose to work with. Ready made plots can be found for the visible region.

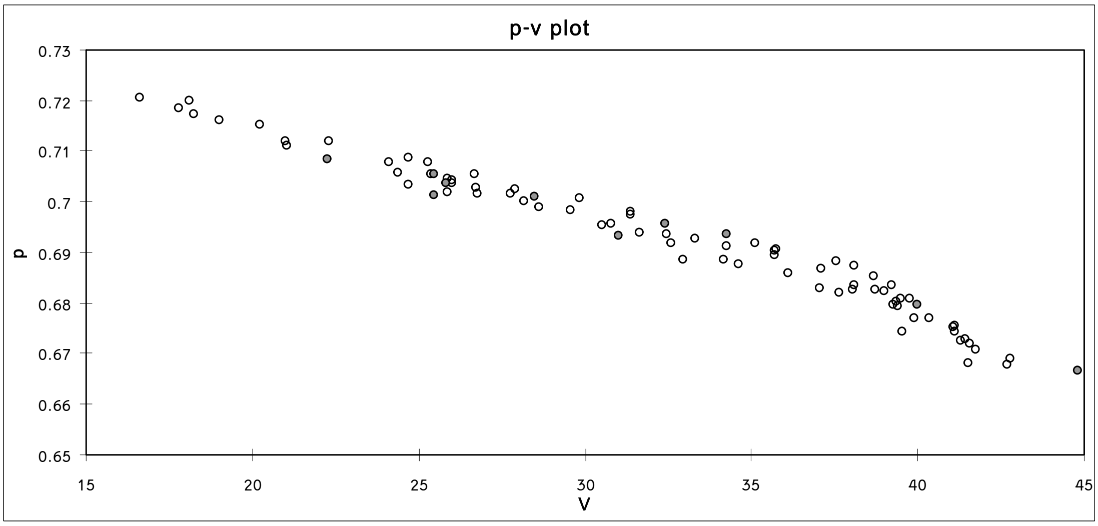

Fig 4: p-v plot. Each glass material is represented by a point

We use the plot to choose pairs of glass materials according to the following guidelines:

- For a small Δp the two glass materials should be on the same horizontal line.
- For a large Δv we want the two glass materials to be as far from each other as possible on this horizontal line.

The choice is not easy since most glass materials reside close to a line that crosses the plot diagonally.

<u>Linear compound prism</u>

Our aim is not to reach an achromatic prism, but rather to have a linearly varying output angle. For that we need to make a change in the design. For the compound prism, equation 3.9 gave us the apex angles of the two glass materials. The results were simulated using Zemax Optical Design Program © Zemax Corp. By changing the apex angle of one of the glass materials we stray from the achromatic design depicted in fig 3. The manner in which this gradually happens may be seen in fig. 5. When the apex angle of the SF11 prism is 3.92° the angular dispersion is quite linear with relation to the wavelength over a very broad spectral region.

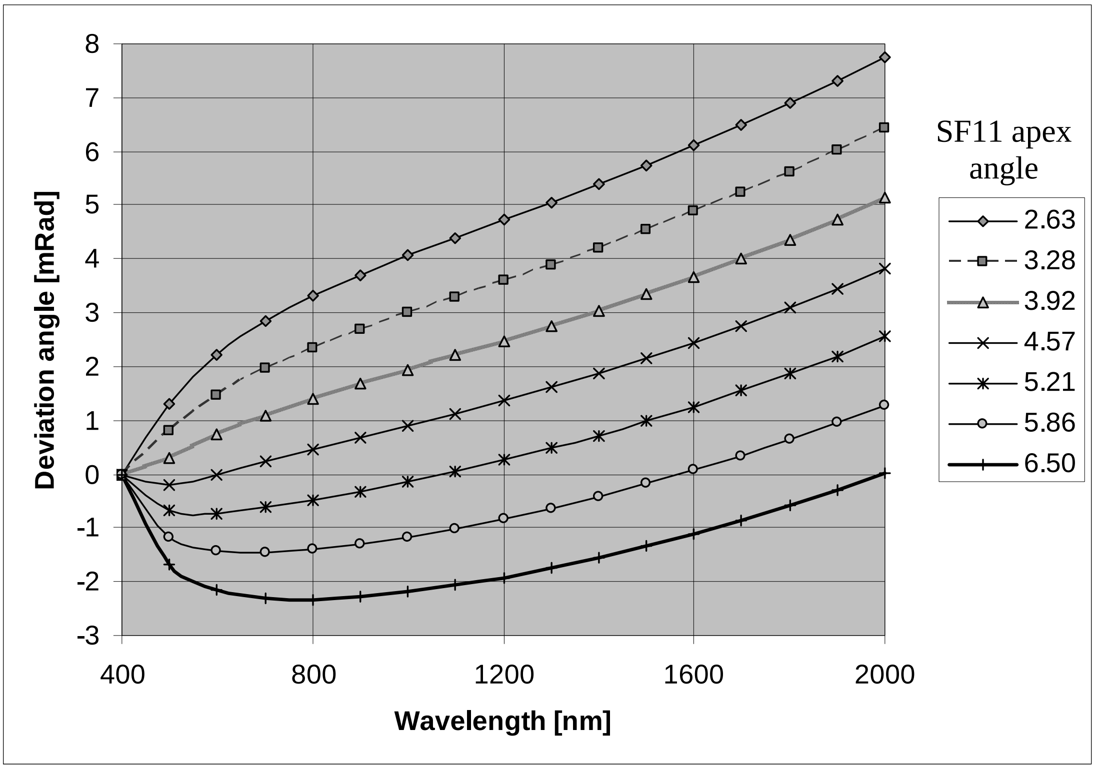

Fig. 5: A 20° BK7 prism inversely coupled to an SF11 prism with apex angle ranging from 6.5° (the calculated apex angle) to 2.6°

An important observation is that the dispersion from a linear compound prism is smaller than the dispersion of a simple prism. In fig. 1 we saw that a 20° BK7 prism gives a dispersion of 8.6mRad (between 400 and 1000nm), while the BK7-SF11 pair in its linear form gives only 1.9mRad – a factor of 22%. These prisms behave similarly for larger apex angles, with dispersion angles growing somewhat faster than linearly.

It should be noted that in order to design the linear compound prism, we wouldn't want a small Δp, but actually a large one. The larger the Δp, the more dispersion we will have. This makes it easier to choose the pair of glass materials since we can simply select two glass materials from both ends of the plot in fig. 4.

Choosing an irregular material such as KDP with SF11 gives a very large Δp, and the resulting dispersion of 4.7mRad may be seen in fig 6 (apex angles of 20° and 3.3°).

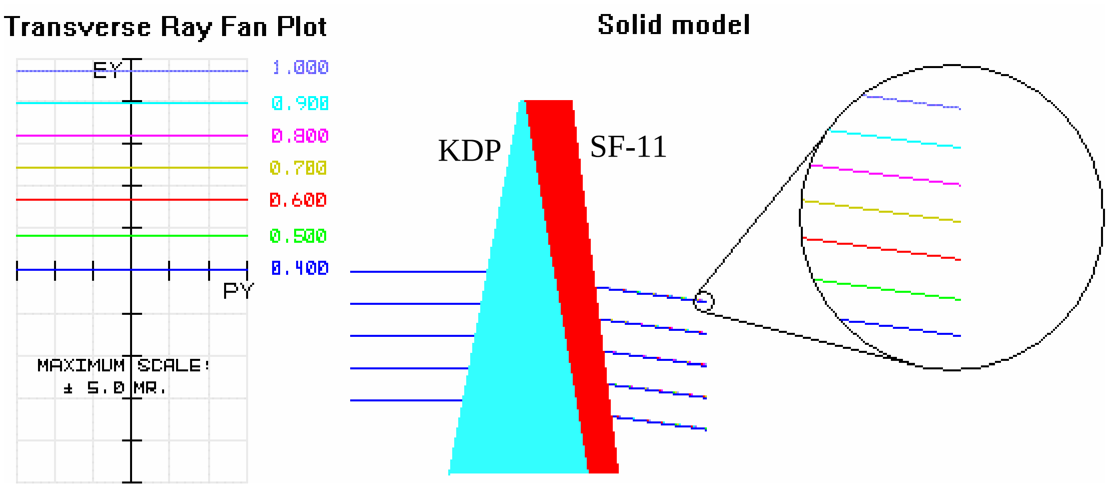

Fig. 6: Linear Dispersion of a 20° KDP prism inversely coupled to a 3.3° SF11 prism